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This report documents the development of of a number of techniques for the computer aided analysis of quasi-optical power combining systems. The major results of the project were the development of models for IMPATT diodes suitable for harmonic balance analysis and transient analysis; the development of quasioptical Green's functions for cavity resonators and grid systems; the development of a Method of Moments code which can use the mixed scalar potential and vector Green's function developed for quasi-optical systems; a transient analysis engine for quasi-optical systems and develeopments of an harmonic balance techniquie suitable for quasi-optical systems. The work makes a broader contribution to the computer aided engineering of microwave and millimeter wave systems with complex device-field interactions.

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CIRCUIT-LEVEL MODELING AND COMPUTER AIDED DESIGN TOOLS FOR MILLIMETER-WAVE QUASI-OPTICAL SYSTEMS

FINAL REPORT

December 10, 1995

U.S. ARMY RESEARCH OFFICE

DAAL03-89-D-0003

NORTH CAROLINA STATE UNIVERSITY

FOREWORD

This report documents the development of of a number of techniques for the computer aided analysis of quasi-optical power combining systems. The major results of the project were the development of models for IMPATT diodes suitable for harmonic balance analysis and transient analysis; the development of quasioptical Green's functions for cavity resonators and grid systems; the development of a Method of Moments code which can use the mixed scalar potential and vector Green's function developed for quasi-optical systems; a transient analysis engine for quasi-optical systems and developments of an harmonic balance technique suitable for quasi-optical systems. The work makes a broader contribution to the computer aided engineering of microwave and millimeter wave systems with complex device-field interactions.

Contents

1 Statement of Problem Studied

In quasi-optical power combining systems a spatial distribution of amplifiers or oscillators and radiating elements are used to produce large amounts of power. The spatial distribution of the amplifiers and their radiating elements is such that the amplifiers are distributed by a substantial fraction of a wavelength. The power from the radiating elements is combined in free-space over a distance of many wavelengths to channel power into a single paraxial mode. The systems used as models in this work are shown in Figures 1 and 2. Quasioptical amplifier and oscillator operation involves complex device-field interaction. Computer aided simulation and subsequent computer aided design (CAD) requires accurate modeling of the field structure and a strategy to integrate the field characterization into a nonlinear circuit simulator. The nonlinear simulator itself must be capable of handling perhaps hundreds of nonlinear devices.

Current commercial microwave computer aided engineering (CAE) packages are oriented towards the design and analysis of planar microwave circuits including hybrid circuits and MMIC's. The computer aided analysis of these circuits is characterized by there being relatively few active devices and by the planar circuit configuration. In nearly all situations the fields are considered as having a secondary effect to that which can be modeled by circuit elements alone. Full field simulations are also used but the problem is simplified by the circuitry of interest being confined to one or two planes. Conventional microstrip Greens functions can be used in an efficient $2\frac{1}{2}$ method of moments-based field solver. In contrast, in a quasi-optical system the interaction of fields with active devices and with planar circuits is inherent to system operation. Furthermore quasi-optical systems rely on combining the powers of numerous closely coupled active devices. Full field simulators have been used to analyze individual cells of a quasioptical system. Analyzing a complete quasioptical system using a 3D simulator is prohibitive in terms of memory and time as a result of the volume discretization required over many tens of wavelengths. The CAE of quasioptical systems requires modeling and analysis techniques which are not supported by existing commercial microwave CAE programs. The purpose of the work proposed here is to develop such a modeling and analysis approach to support the design and development of quasi-optical systems. The fundamental issues here are

1. handling device-field interactions in a non-planar environment,

2. handling a very large number of active devices in steady-state harmonic balance analysis.

3. handling distributed high Q passive components in transient analysis.

The low efficiencies achieved with current quasioptical system design methodologies is, presumably, because of poor impedance matching of the device to the circuit in which it is impeded and perhaps interaction with surrounding active devices. Better CAE tools are required. The aim of this project was to develop these required CAE technologies for quasi-optical systems. Closely allied with this was the development of verification systems. An open cavity oscillator, shown in Figure 1, and a two dimensional dielectric quasioptical system, shown in Figure 3, were developed.

2 Summary of Most Important Results

Contributions were made to the computer aided engineering of quasi-optical systems that were realized in this project are

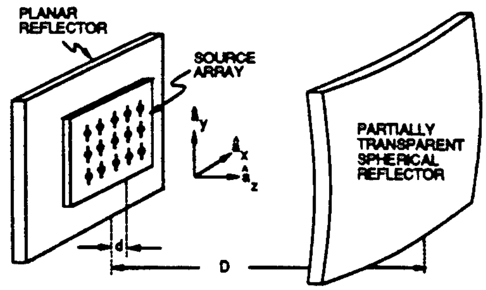

Figure 1: Cross-section of the plano-concave open resonator.

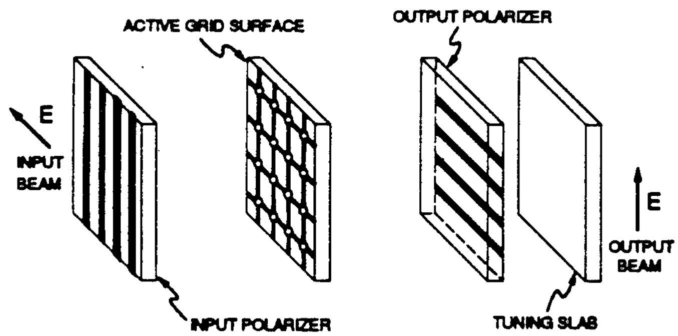

Figure 2: Grid amplifier with X and Y polarizers.

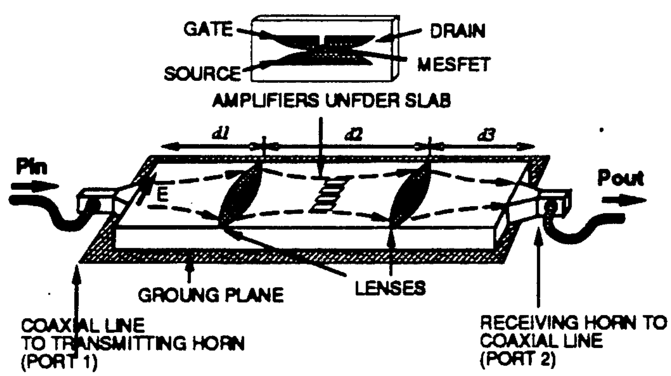

Figure 3: Grid amplifier with X and Y polarizers.

1. Development of quasi-optical specific Green's functions. See section 2.1.

2. Development of a method of moments field analysis code which uses the quasi-optical Green's functions. See section 2.2.

3. Development of an X-band open cavity quasi-optical power combining oscillator. See section 2.3.

4. Development of a steady-state (harmonic balance) analysis engine for quasi-optical systems. See section 2.4.

5. Development of a transient analysis engine for high Q quasi-optical systems. See section 2.5.

6. Development of an enhanced model for and measurement based modeling technique for an IMPATT diode. See section 2.6.

2.1 Quasi-optical Green's Function Development

The approach that we used considered the total field radiating from a device as a combination of paraxial fields and nonparaxial fields. The paraxial fields are quasi-optical fields described by Hermite-Gaussian or Laguerre-Gaussian modes. The nonparaxial fields are described by a free space or perhaps half space Green's function. In more complicated situations with substrates the free space or half space Green's function could be replaced by the appropriate layered Green's function. The trick is that the paraxial "free-space fields" are counted twice. With the double accounting removed the complete field description is obtained. The quasioptical component of the fields describes global coupling through the cavity and the nonparaxial fields describe near neighbor coupling via direct radiation.

Details of this work are given in the following publications:

- P.L. Heron, J.W. Mink, G.P. Monahan, F.W. Schwering and M.B. Steer, "Impedance matrix of an antenna array in a quasi-optical resonator," *IEEE Trans. Microwave Theory Techniques*, Vol. 41, October 1993, pp. 1816-1826.

  The power from numerous millimeter wave solid-state sources can be efficiently combined using quasi-optical techniques. One such technique is to place an array of active radiating sources within a quasi-optical resonator. The driving point impedance of each antenna is strongly effected by the presence of all other active antennas as well as by the mode structure and Q of the resonator. In this paper the impedance matrix for an array of antennas radiating into a plano-concave open resonator is determined through use of the Lorentz integral. The resulting expressions include the effect of diffraction loss and are valid for arbitrary reflector spacing, source frequency, array location and geometry. The result can be used to impedance match each active source to its antenna and thus facilitate design of an efficient power combining system. Simulations using the impedance matrix in conjunction with an antenna impedance model are compared with two-port measurements.

- P.L. Heron, F.W. Schwering, G.P. Monahan, J.W. Mink, and M.B. Steer, "A dyadic Green's function for the plano-concave quasi-optical resonator," *IEEE Microwave and Guided Wave Letters*, Vol. 3, August 1993, pp. 256-258.

Quasioptical power combining using an open resonator is an efficient and robust means to combine the power of numerous oscillators. Here an approximate dyadic Green's function is derived for a quasi-optical resonator. The resonator consists of an infinite perfectly conducting planar reflector facing a partially reflecting concave spherical surface of finite aperture. and includes the effects of direct radiation loss, mode diffraction losses, and conductor losses. Near field effects are included so that direct radiation coupling between antennas can be accurately determined.

- Pat Heron, *Design of Millimeter Wave Quasi-Optic Power Combiners*, Ph.D. Dissertation, North Carolina State University, 1993.

Sponsored research was directed toward developing millimeter wave power sources utilizing quasi-optical techniques. A system consisting of an array of oscillators that radiated into a quasi-optical resonator was analyzed. Each oscillator was comprised of a solid state device and a radiating structure. A dyadic Green's function was developed for a Fabry-Perot resonator which consisted of a metallic planar reflector and a shallow spherical metallic reflector. The Green's function was applied to determine the driving point impedance matrix for an array of electrically small antennas within the resonator. An experimental X-band resonator was designed and fabricated, then one and two-port measurements were used to validate the theoretical calculations. A technique was determined for simulation of antennas that are not electrically small which radiate into the cavity. These techniques are shown to be applicable to coupling structures for quasi-optical systems in general.

Practical considerations regarding the simulation of nonlinear solid state driving elements were addressed. A a technique for efficient Jacobian calculation using the multidimensional fast Fourier transform as well as a technique for simulator time-domain oversampling were developed so that multiple oscillator systems can be efficiently simulated. These simulation techniques were implemented and tested in a harmonic balance circuit simulator.

- P.L. Heron, G.P. Monahan, F.W. Schwering, J.W. Mink, and M.B. Steer "Multiport circuit model of an antenna array in an open quasi-optical resonator," Proc. 1993 URSI Conference, June 1993, p. 84.

In a quasi-optical power combiner the output of many solid-state sources is efficiently and robustly combined. This is achieved as a result of the one-to-many coupling as the power radiated by a single element is reflected by the curved reflector on to many oscillators. The driving point impedance of each antenna is strongly effected by the presence of all other active antennas as well as by the mode structure and Q of the resonator. In this presentation the derivation of a field theoretic model describing the coupling is described. An impedance matrix is determined through use of the Lorentz reciprocity theorem and calculation of the aperture and conductor losses. A half space Green's function is used to incorporate the near-field effect of non quasi-optical fields. The result obtained can be used to impedance match each active source to its antenna and thus facilitate design of an efficient power combining system. As an example the calculated and measured transimpedance of two antennas are considered.

- P.L. Heron, G.P. Monahan, J.E. Byrd, M.B. Steer, F.W. Schwering and J.W. Mink, "Circuit level modeling of quasioptical power combining open cavities," *1993 IEEE MTT-S International Microwave Symposium Digest*, June 1993, pp. 433-436.

A multiport circuit level model is developed for an open-cavity, quasioptical power combiner. The model is developed using Hermite-Gaussian beam mode theory, the Lorentz reciprocity theorem and determination of diffraction losses. An impulse response/convolution-based technique being developed for incorporating this high Q model in a transient simulator is discussed.

2.2 Method of Moments Analysis

All other schemes for handling the fields in quasi-optical systems use a unit cell approach where ideal magnetic and electric current walls are used. The unit cell based modeling approach is akin to the circuit element modeling in conventional microwave simulators. While this leads to fast circuit simulation compromises are made. This Green's function uses a mixed paraxial/nonparaxial Green's function approach which is combined with an extremely efficient Conventional Method of Moments code using uses a scalar Green's function whereas the quasioptical Green's function is a mixed scalar potential and vector Green's function. It would be prohibitively complex to develop a scalar potential Green's function for a quasi-optical system including effects such as diffraction losses and conductive losses. However the technique that we developed supports the heuristic development of Green's functions and not just for quasi-optical systems. This could lead to important developments in the top-down design of microwave and millimeter-wave systems in that an approximate, heuristically-based, Green's function could be used in early system design.

Details of this work are given in the following publications:

- T.W. Nuteson, G.P. Monahan, M.B. Steer, K. Naishadham, J.W.Mink, K. Kojucharoff and J. Harvey, "Full-wave analysis of quasi-optical structures," *IEEE Trans. Microwave Theory Techniques*, 1996, *In Press.*

  A full-wave moment method implementation, using a combination of spatial and spectral domains, is developed for the analysis of quasi-optical systems. An electric field dyadic Green's function, including resonant and non-resonant terms corresponding to coupling from modal and non-modal fields, is employed in a Galerkin routine. The dyadic Green's function is derived by separately considering paraxial and non-paraxial fields and is much easier to develop than a mixed, scalar and vector, potential Green's function. The driving point impedance of several antenna elements in a quasi-optical open cavity resonator and a 3×3 grid in free space are computed and compared with measurements.

- T.W. Nuteson, G.P. Monahan, M.B. Steer, K. Naishadham, J.W. Mink, and F.K. Schwering, "Use of the moment method and dyadic Green's functions in the analysis of quasi-optical structures," *1995 IEEE MTT-S International Microwave Symposium Digest*, May 1995. pp. 913–916.

  A moment method using a dyadic Green's function is developed for the analysis of quasi-optical systems. The dyadic Green's function used has separate terms for the paraxial and non-paraxial fields and is much easier to develop than a mixed potential Green's function. The method is applied to the analysis of antenna elements in a quasi-optical resonator.

2.3 X-band Open Cavity Quasi-Optical Power Combining Oscillator

An X-band open cavity quasi-optical oscillating power combining system, as shown in Figure 1, was constructed as one of the verification systems. The oscillator used IMPATT diodes but the

results were disappointing.

Details of the work are described in

- Gregory Monahan, *Characterization and Design of Millimeter Wave Quasi-Optical Power Combiners*, Ph.D. Dissertation, North Carolina State University, 1995.

  Research was directed towards developing an experimentally based design methodology for quasi-optical power combining using IMPATT diode oscillators. The design and characterization of an open cavity resonator for use in a quasi-optical power combiner is presented. The lowest order Hermite Gaussian and Laguerre Gaussian modes of Fabry-Perot (open cavity) resonators are shown to be degenerate. A coaxial test fixture was designed, fabricated and used to experimentally characterize post mounted IMPATT diodes in the small signal regime. This information was then used to design a radiating oscillator operating at 8.65 GHz consisting of an IMPATT diode connected directly to a rectangular patch antenna. The operation of the oscillator in free space and in the presence of cavity modes was characterized. The effect of the cavity modes upon the operation of the oscillator was explored. An array of three IMPATT diode oscillators operating in the open cavity resonators was developed using the design methodology presented herein. Design, construction, instrumentation, measurements, and data presentation of the experimental apparatus is described.

- G.P. Monahan, P.L. Heron, M.B. Steer, J.W. Mink and F.W. Schwering "Mode degeneracy in quasi-optical resonators," *Microwave and Optical Technology Letters*, Vol. 8, No. 5, April 5 1995, pp. 230-232.

  Fabry-Perot resonators and beam waveguides are used in quasi-optical systems to refocus electromagnetic fields. These fields are described by Laguerre Gaussian or Hermite Gaussian quasi-TEM mode families dependent on whether the aperture is rectangular or circular. It is shown that certain lower order Laguerre Gaussian and Hermite Gaussian modes are identical. This has implications for the design of quasi-optical systems for mode selection.

2.4 Steady-State (Harmonic Balance) Analysis of Quasi-Optical Power Combiners

Modifications were made to the conventional harmonic balance algorithm to support active quasi-optical systems. The analysis engine incorporates novel memory and analysis efficiencies so that the analysis of 100's of active devices in a quasioptical system is possible. Procedures for the optimization of power added efficiency and output power, subject to stable optimization, will be developed. Currently all of the elements must be assumed to oscillate at the same frequency. The CAD tool enables the following to be investigated: the phase distribution across the antenna array; power and oscillation frequency.

Background for the work is given in the following publications:

- M.B. Steer and S.G. Skaggs, "CAD of GaAs microwave circuits: historical perspective and future trends," invited paper, Korea-United States Design and Manufacturing Workshop, November 17-19, 1993, Taejon, Korea, pp. 109-118.

  The nonlinear analysis of microwave circuits has reached a high level of maturity over the last decade. By assuming that only a finite number sinusoids are present in a nonlinear circuit, the computational burden of computing the transient response of the circuit is avoided and

only the steady state response, given by the amplitudes and phases of the sinusoids, is required. The Harmonic Balance technique is ideally suited to solving for the phasors of the steady state response. However this technique is not as versatile as transient analysis techniques. An historical perspective is presented and future trends and requirements for the CAD of GaAs microwave circuits are outlined.

- M.B. Steer and S.G. Skaggs, "CAD of GaAs microwave circuits: historical perspective and future trends," invited paper, Korea-United States Design and Manufacturing Workshop, November 17-19, 1993, Taejon, Korea, pp. 109-118.

Details of this work that have not been published elsewhere are given in Appendix A.

2.5 Transient Analysis of a Quasi-Optical Power Combiner

Current design and operation concerns of quasi-optical combining systems largely involve locking behavior. Much of the concept behind quasi-optical power combining relies on global locking of the various oscillating elements. However maximizing the power extracted from the system requires that the elements be placed relatively closely to each other. This increases the nearest element interactions so that local locking rather than global locking becomes dominant. However some local locking is required to ensure that the sources initially lock. To model these effects transient simulation is required. We have used an existing convolution-based transient simulator to successfully model a simple model of a single element quasi-optical cavity oscillator. The simulation strategy uses the impulse response of the linear circuitry to interface the frequency-dependent cavity model to the nonlinear oscillating element model. Interfacing is achieved through convolution.

Transient analysis is essential in determining some of the most critical parameters of a quasioptical systems. Locking behavior and 0Φoccurrence of chaotic behavior cannot be determined any other way. However transient analysis of steady state microwave systems is notoriously slow. Far too slow for design optimization. In conventional microwave circuit design the lengthy simulation time is reduced through application of harmonic balance analysis. Simulation time reduction by a factor of a thousand (and very often much more) is obtained. It then becomes feasible to apply circuit optimization techniques to maximize power added efficiency, output power and other design parameters.

We have adapted a technique developed in another project (M.S. Basel, M.B. Steer and P.D. Franzon "Simulation of High Speed Interconnects Using a Hierarchical Packaging Simulator," *IEEE Trans. on Components Hybrids and Manufacturing Technology/Advanced Packaging*, February 1995, pp. 74-82.) for the transient analysis of distributed circuits and in particular for high Q quasi-optical systems. The following describes what was done: The transient analysis of a distributed circuit without a canonical model requires the use of Fourier transform techniques to translate the frequency domain characterization of the linear circuit to an impulse response that can be used in a transient convolution-based analysis. This transformation would normally lead to significant aliasing for high Q circuits. In the work by Basel et al. we presented a method which preserves the characterization in generating a modified impulse response. The modification of the impulse response is removed during iteration. Prototype code for performing the analysis of a quasioptical system has been completed.

In transient analysis either a canonical model of the distributed quasioptical system must be developed or else an impulse response derived by Fourier transforming a very large number of

model evaluations at discrete frequencies. Modeling compromises are made in both cases. The canonical modeling approach models the complicated distributed circuit (the quasioptical system) by a lumped element model In the impulse response approach which we use the aliasing effects can never be entirely eliminated and many more model evaluations will invariably be required than that required in an harmonic balance analysis.

Details of this work are mostly given in the paper by Basel et al. Prototype code has been written but the code is not compatible with the Spice simulation approach. This will be addressed in the subsequent commercialization activities.

Background and description of the work conducted early in this project are described in the following publication:

- M.B. Steer, "Simulation of microwave and millimeter-wave oscillators, present capability and future directions," Proc. 1992 Workshop on Integrated Nonlinear Microwave and Millimeter-wave Circuits, October 1992 (invited keynote paper).

  The current state-of-the art of oscillator simulation techniques is presented. Candidate approaches for the next generation of oscillator simulation techniques are reviewed. The method is presented which uses an efficient and robust convolution-based procedure to integrate frequency-domain modeling of a distributed linear network in transient simulation. The impulse response of the entire linear distributed network is obtained and the algorithm presented herein ensures that aliasing effects are minimized by introducing a procedure that ensures that the interconnect network response is both time-limited and band-limited. In particular, artificial filtering to bandlimit the response is not required.

Details of the work that have not been published elsewhere are given in Appendix B.

2.6 IMPATT Device Model

a The open cavity power combining oscillator used IMPATT diodes as the active device and it was anticipated that an existing IMPATT diode model could be used in computer simulations. However the existing models do not conserve charge when they are operated in the large signal regime. As such the development of large signal IMPATT diode models suitable for transient analysis and harmonic balance analysis were developed. Extensive active device modeling was not anticipated in this project.

Details of this work that have not been published elsewhere. are given in the Appendix C

Experimental IMPATT Characterization

Details of the experimental IMPATT characterization work are given in the following publications:

- M.B. Steer, "Diode characterization in a coaxial mount," *International Journal of Microwave and Millimeter Wave Computer Aided Engineering*, Vol. 3, April 1993, pp. 114-117.

  A previously presented technique for experimentally characterizing packaged microwave diodes mounted in a coaxial test fixture is investigated. It is demonstrated that the fields surrounding the diode and the calibration packages are adequately developed to provide an accurate diode equivalent circuit model for use computer aided design.

- G.P. Monahan, A.S. Morris and M.B. Steer, "A coaxial test fixture for characterizing low impedance microwave two-terminal devices," *Microwave and Optical Technology Letters*, Vol. 6, March 1993, pp. 197-200.

A coaxial test fixture is designed and used to measure the input impedance of a packaged IMPATT diode in the frequency range from 0 to 18 GHz. In this paper such a mount is presented which incorporates a coaxial transformation to transform the relatively high measurement characteristic impedance to low impedance of a coaxial line with an oversized center conductor. Following the coaxial transformer the fields are converted to a radial structure corresponding to the fields of a mounted diode. This diode characterization procedure is being used to develop IMPATT diode models for use in the simulation of a quasioptical power combining oscillator wherein the diode appears under a sub-resonant patch antenna. The work is an extension of our earlier work but with greater attention paid to the coaxial transformer.

2.7 Two-Dimensional Quasi-Optical Power Combiner

- H. Hwang, T.W. Nuteson, M.B. Steer, J.W. Mink, J. Harvey and A. Paolella, "Quasi-optical power combining in a dielectric substrate, International Symposium on Signals, Systems and Electronics, October 25–27, 1995, pp. 89-92.

  This paper presents a quasi-optical MESFET-based dielectric slab waveguide with an amplifier array. Up to 19 dB amplifier gain and 9.5 dB system gain is obtained at 7.38 GHz. Measurements of amplifier and system gain, $|S_{21}|$, output power vs. input power and transverse power distribution are presented.

- H. Hwang, G.P. Monahan, M.B. Steer, J.W. Mink, and F.K. Schwering, "A dielectric slab waveguide with four planar power amplifiers," *1995 IEEE MTT-S International Microwave Symposium Digest*, May 1995. pp. 921–924

  A hybrid dielectric slab beam waveguide with four MESFET amplifiers employing quasi-optical power combining is reported for the first time. Up to 10 dB power gain is obtained at 7.4GHz. Measurements for power gain, amplifier gain, insertion loss and transverse power distribution are presented. The fabrication technique employed is suitable for planar MMIC circuits.

- F. Poegel, S. Irrgang, S. Zeisberg, A. Schuenemann, G.P. Monahan, H. Hwang, M.B. Steer, J.W. Mink, and F. K. Schwering, "Demonstration of an oscillating quasi-optical slab power combiner," *1995 IEEE MTT-S International Microwave Symposium Digest*, May 1995. pp. 917–920.

  Power combining in a hybrid dielectric slab-beam waveguide resonator using a MESFET oscillator array is reported for the first time. Four MESFET oscillators lock via quasi-optical modes to produce a signal at 7.4 GHz with a 3 dB linewidth of less than 3 kHz.

- A. Schuneman, S. Zeisberg, P. L. Heron, G. P. Monahan, M. B. Steer, J. W. Mink, F. W. Schwering, "A prototype quasi-optical slab resonator for low cost millimeter-wave power combining," *Workshop on Millimeter-Wave Power Generation and Beam Control*, Huntsville, Alabama, September 14 – 16, 1993, pp. 235-243.

  A quasi-optical slab resonator is a planar version of an open quasi-optical resonator and has many potential manufacturability and cost advantages. It uses a waveguiding mechanism that combines beam-waveguide and dielectric waveguide propagation modes to confine fields

to a plane. Here the slab resonator is experimentally characterized and design data presented for a prototype quasi-optical resonator.

- J.W. Mink, F.W. Schwering, P.L. Heron, G.P. Monahan, A. Schuneman, S. Zeisberg, and M.B. Steer, "A hybrid dielectric slab-beam waveguide, theory and experiment," *19 th Army Science Conference*, June 1994.

- A. Schuneman, S. Zeisberg, G.P. Monahan, F.W. Schwering, J.W. Mink, and M.B. Steer, "Experimental investigation of a quasioptical slab resonator," Proc. 1993 URSI Conference, June 1993, p. 27.

  Quasi-optical power combiners using open-resonators achieve efficient and robust combining at millimeter-wave frequencies and above. However systems cannot be photolithographically defined thus limiting mass production and contributing to high cost. A similar structure more amenable to photolithographic reproduction is the hybrid dielectric slab-beam waveguide (HDSBW). This structure combines the waveguiding principles of dielectric surface waves of a slab guide surface and the confined beam corresponding to Gauss-Hermite beam modes. In this paper a HDSBW slab resonator was experimentally investigated.

- S. Zeisberg, A. Schunemann, G.P. Monahan, P.L. Heron, M.B. Steer, J.W. Mink and F.W. Schwering, "Experimental investigation of a quasi-optical slab resonator," *IEEE Microwave and Guided Wave Letters*, Vol. 3, August 1993, pp. 253-255.

  A quasi-optical slab resonator for TE modes was experimentally characterized to demonstrate a planar technology for quasi-optical devices. Predicted and measured frequencies of resonance of the TE slab modes and electric field profiles are in close agreement.

- H. Hwang, T.W. Nuteson, M.B. Steer, J.W. Mink, J. Harvey and A. Paolella, "Slab-based quasi-optical power combining system," Infrared and Millimeter-Wave Conference, Orlando Florida, December 1995.

  A slab-based quasi-optical power combining system with convex and concave lenses is investigated. Experimental results imply that a concave-lens system has less scattering loss and higher system gains than a concave-lens system. An amplifier gain of 15 dB and a system gain of 8 dB were achieved.

- H. Hwang, T.W. Nuteson, M.B. Steer, J.W. Mink, J. Harvey and A. Paolella, "Quasi-optical power combining techniques for dielectric substrates," *International Semiconductor Device Research Symposium*, December 6-8, 1995.

  The dielectric slab system described here has the advantage of being two-dimensional and is thus more amenable to photolithographic reproduction than the conventional open quasi-optical power combining structures. The previous investigations of the quasi-optical dielectric slab cavity and waveguide demonstrated the suitability of this structure for the integration of quasi-optical power combining with MMIC technology. A complete DSBW quasi-optical system, as shown in Fig. 1, could consist of the following: a source, active or injection, an amplifier array, triplers, and a leaky wave antenna which would be used for steering the energy out of the system. Between each of the stages lenses are used to focus the guided waves for optimal field concentration on the elements in the system. In this work we present the amplifier array stage using both convex and concave lenses as shown

in Fig. 2. The DSBW amplifier system incorporates four MESFET amplifiers and two thin convex/concave lenses. The waveguide system was adjusted with the transistors turned off so that the guided waves are focused near the aperture of the receiving horn. The dielectric slab is Rexolite ($\epsilon = 2.57$, $\tan\delta = 0.0006$ at X-band), and it is 27.94 cm wide, 62 cm long, and 1.27 cm thick. The convex lenses are fabricated from Macor ($\epsilon = 5.9$, $\tan\delta = 0.0025$ at 100 kHz) with a radius of 30.48 cm, and the focal length, f, is 28.54 cm. The concave lenses are air ($\epsilon = 1$) with a radius of 30.48 cm, and the focal length, f, is 40.4 cm. The aperture width of both horn antennas is 9 cm, designed to be wide enough to catch most of the amplified power. Energy emitted from the input radiator propagates in a quasi-optical TE Gaussian mode in the dielectric slab waveguide,and is focused by the first lens in the middle area of the slab. This system is designed so that the amplifier unit cells are within the beam waist (the $1/e$ field points).

3 List of Publications Resulting from this Project

1. T.W. Nuteson, G.P. Monahan, M.B. Steer, K. Naishadham, J.W.Mink, K. Kojucharoff and J. Harvey, "Full-wave analysis of quasi-optical structures," *IEEE Trans. Microwave Theory Techniques*, 1996, *In Press.*

2. H. Hwang, T.W. Nutesson, M.B. Steer, J.W. Mink, J. Harvey and A. Paolella, "Slab-based quasi-optical power combining system," Infrared and Millimeter-Wave Conference, Orlando Florida, December 1995.

3. H. Hwang, T.W. Nutesson, M.B. Steer, J.W. Mink, J. Harvey and A. Paolella, "Quasi-optical power combining techniques for dielectric substrates," *International Semiconductor Device Research Symposium*, December 6-8, 1995, *In Press.*

4. H. Hwang, T.W. Nutesson, M.B. Steer, J.W. Mink, J. Harvey and A. Paolella, "Quasi-optical power combining in a dielectric substrate, International Symposium on Signals, Systems and Electronics, October 25–27, 1995, pp. 89-92.

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List of dissertations resulting from this project

1. Gregory Monahan, Characterization and Design of Millimeter Wave Quasi-Optical Power Combiners, Ph.D. Dissertation, North Carolina State University, 1995.
2. Pat Heron, Design of Millimeter Wave Quasi-Optic Power Combiners, Ph.D. Dissertation, North Carolina State University, 1993.

3.1 List of Scientific Personnel Supported by Project

1. Michael B. Steer, principal investigator
2. Patrick L. Heron, received Ph.D. in 1993
3. Gregory P. Monahan, received Ph.D. in 1995
4. Todd W. Nuteson, continuing Ph.D. student
5. Huan-Sheng Hwang, continuing Ph.D. student
6. Steven Skaggs, continuing Ph.D. student
7. Jefferey Byrd, terminated Ph.D. studeies during contract.

4 Future Direction

The work described here is being continued in three projects:

1. "Circuit Modeling and Computer Aided Design for Quasioptical Systems," Scientific Research Associated, Inc. under U.S. Army Missile Command contract DAAH01-95-C-R111.
2. "Computer Aided Engineering Tools for Microwave and Millimeter-Wave Quasi-optical Oscillators and Amplifiers," Compact Software, Inc. under ARPA SBIR Program contract DAAH01-95-C-R203.
3. "Research and Development of a Quasi-Optical Dielectric Slab Power Combining System," U.S. Army Research Office through grant DAAH04-95-1-0536.

The first two projects involve commercialization of the activities conducted under the project reported on here. The third project is continuation of the two-dimensional power combiner work described here and the development of a complete two-dimensional power combining system.

A Harmonic Balance Analysis of Quasi-Optical Power Combining Circuits

Harmonic balance analysis has become the accepted method for finding the steady state response of nonlinear microwave circuits. This analysis requires the solution of a system of nonlinear equations by Newton's method, and is often limited by the size of the Jacobian matrix. For

a large matrix, both memory usage and run time costs can become significant. Φ This papers commercial harmonic balance simulators use different sparsification techniques. The essence of the work we propose here is to merge the sparsification techniques to obtain a efficient harmonic balance simulation for circuits with many active devices.

Harmonic balance analysis is a robust and efficient technique for the analysis of small circuits, but memory demands increase rapidly as problem size increases. Memory usage is a critical factor for very large problems as once RAM is exceeded, disk swapping is used with an associated factor of 10 reduction in speed. In modern harmonic balance programs, in which the elements of the Jacobian are calculated analytically, the model evaluation time is comparable to that required for matrix formulation (R. Gilmore and M.B. Steer, "Nonlinear Circuit Analysis Using the Method of Harmonic Balance — a Review of the Art: Part II, Advanced Concepts" *International Journal on Microwave and Millimeter Wave Computer Aided Engineering*, Vol. 1, April, 1991, pp. 159-180) For large problems, for example with many tones and/or many nonlinear elements, the matrix manipulation time becomes the major component of the simulation time. Also, the size of the Jacobian matrix dominates memory usage. With the aim of reducing both memory utilization and simulation time, conventional and blocked sparse matrix techniques and Samanskii's (chord) method have been used. Each approach to sparsification dramatically reduces memory usage and simulation times. However the memory usage and simulation times are still extremely large for many-device circuits such as Quasioptical amplifiers.

Technically, the modifications require combining the block Newton iteration method (C.R. Chang, P.L. Heron and M.B. Steer "Harmonic Balance and Frequency Domain Simulation of Nonlinear Microwave Circuits using the Block Newton Method," *IEEE Trans. Microwave Theory Techniques*, April 1990, pp. 431-434.) efficient Jacobian calculation (P.L. Heron, and M.B. Steer, "Jacobian calculation using the multidimensional fast Fourier transform in the harmonic balance analysis of nonlinear microwave circuits," *IEEE Trans. Microwave Theory Techniques*, April 1990, pp. 429-431) with the conventional sparse matrix technology. Extensions to detect instabilities leading to chaotic phenomena in such a large system remain to be developed.

A.1 Newton Iteration Procedure

In applying harmonic balance analysis, a system of nonlinear circuit equations is developed and an error function is formulated and minimized. This error function typically involves the Kirchhoff's current law error at the interface between the linear and nonlinear circuits, but may include other error determinants (such as Kirchhoff's voltage law) for those elements that can not be described as controlled current sources. The error function is minimized with respect to a set of state variables to an acceptable level using Newton's method. The state variables are typically the phasors of the voltages at the interface nodes, although the use of alternative state variables dramatically improves convergence in some cases.

The harmonic balance problem can be formulated in an iterative form as follows

$$ {}^{i+1}\mathbf{x} = {}^{i}\mathbf{x} - \alpha \mathbf{J}^{-1}({}^{i}\mathbf{x}) \cdot \mathbf{f}({}^{i}\mathbf{x}). \tag{1}$$

where $\mathbf{f}({}^{i}\mathbf{x})$ the error vector at the i^{th} iterate and ${}^{i}\mathbf{x}$ the vector of state variables at the i^{th} iterate of $\mathbf{f}({}^{i}\mathbf{x})$, the iterative update procedure is given by
The harmonic balance error is defined as the norm of the $\mathbf{f}({}^{i}\mathbf{x})$ vector in (1) and iteration continues until this error is less than some user-defined value.

Various modifications of the Newton update procedure outlined in (1) are possible. Rarely is

the inverse of the Jacobian evaluated directly. Instead the corresponding matrix equation

$$\mathbf{J}(^{i}\mathbf{x})(^{i+1}\mathbf{x} - ^{i}\mathbf{x}) = \mathbf{f}(^{i}\mathbf{x}) \tag{2}$$

is solved for $^{i+1}\mathbf{x}$ using LU decomposition. The $^{i+1}\mathbf{x}$ estimate is then adjusted by the scaling factor α. From the solution of (2) $\mathbf{J}^{-1}(^{i}\mathbf{x})$ can be evaluated. Furthermore, in Samanskii's modification of the procedure, $\mathbf{J}^{-1}(^{i}\mathbf{x})$ determined at one iteration is reused at subsequent iterations as long as the harmonic balance error decreases. In practice $\mathbf{J}(\mathbf{x})$ is a sparse matrix and many of the entries are relatively small. Through ordering of the rows and columns of the Jacobian, the significant entries form blocks along the diagonal with only relatively small entries outside these blocks. By ignoring the out-of-block entries the Jacobian becomes a block-diagonal Jacobian which can be manipulated at a fraction of the memory and time costs that a full Jacobian can be manipulated. The sparsity can be taken advantage of using conventional sparse matrix representation resulting in dramatic reductions in memory usage and increases in computation speed. The performance improvements obtained using different combinations of sparse matrix, blocking, and Samanskii's method are the subject of this paper. Greater specifics are discussed below in the context of the specific implementation.

A.2 Results to Date

Aspects of the proposed approach have been implemented in a prototype simulation program called TRANSIM. First, TRANSIM forms the modified nodal admittance matrix of the circuit, which is used to obtain the linear response to each iterate of $\mathbf{x}$. Next, the interface unknowns are transformed to the time domain and applied to the nonlinear elements to determine the nonlinear response and error vector. If the resulting error is less than the error of the previous iterate, the Jacobian is calculated. Otherwise, the scaling factor α is reduced and the same Jacobian is reused. The time domain derivatives are converted directly to frequency domain derivatives by use of a multidimensional fast Fourier transform(NFFT) technique.

As seen in (1), the term $\mathbf{J}^{-1}(^{i}\mathbf{x}) \cdot \mathbf{f}(^{i}\mathbf{x})$ must be found. Instead of performing the inversion of $\mathbf{J}$, TRANSIM solves the linear equation $\mathbf{J}(^{i}\mathbf{x}) \cdot b = f(^{i}\mathbf{x})$ by LU factorization of $\mathbf{J}(^{i}\mathbf{x})$. Since the LU factorization of a matrix $\mathbf{A}$ need be computed only once to solve any equation $\mathbf{A}x = b$, LU factorization can help reduce run time when the Jacobian is reused.

Perhaps the simplest way to reduce run time is the method proposed by Samanskii, also called the chord method. Since the Jacobian often does not change significantly between iterations, it is not always necessary to recalculate the Jacobian at each step. Instead, the same Jacobian may be used for more than one iteration. In fact, for weakly nonlinear circuits at low power, the Jacobian may need to be calculated only once. For more strongly nonlinear circuits, some criterion may be determined for how often the Jacobian is updated. TRANSIM uses the same Jacobian for each iteration until the error has been improved by less than 5 percent since the last iterate. At that point, the Jacobian is recalculated.

Another method for reducing run time is the block Jacobian method. For many nonlinear circuits, the Jacobian may be arranged so that most of the nonzero elements are located in blocks on the diagonal. Instead of solving one set of N equations, one may solve M sets of P equations, where M is the number of diagonal blocks and $MP = N$. For this method, (1) becomes:

$$^{i+1}\mathbf{x}_k = ^{i}\mathbf{x}_k - \alpha \mathbf{J}_k^{-1}(^{i}\mathbf{x}) \cdot \mathbf{f}(^{i}\mathbf{x}) \qquad k = 0, 1, \cdots, M. \tag{3}$$

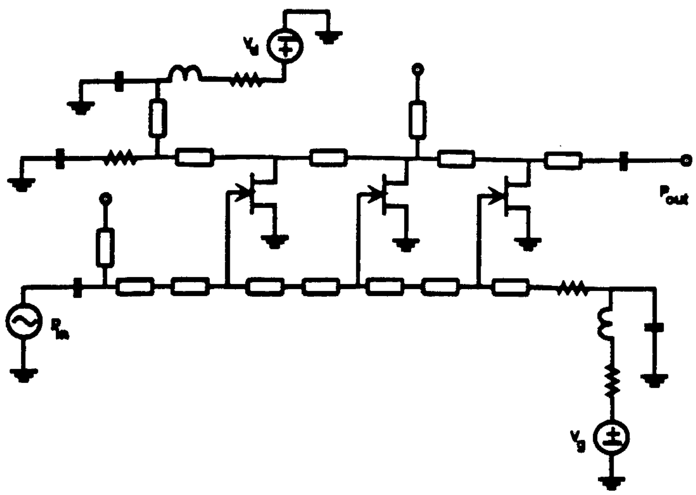


Figure 4: Schematic of the distributed amplifier.

Since the number of operations required to solve N equations is proportional to $N^3/3$, it is clear that much computation time can be saved.

The problem of memory usage is most effectively addressed by the use of sparse matrices. For many problems, the Jacobian is quite sparse, i.e., there are many entries in the matrix which are zero. Rizzoli et al. describe a method for assuming a pattern of zero entries in the Jacobian and using specialized matrix solvers, but the entire matrix still must be allocated. A sparse matrix package such as the one developed by Kundert and implemented in SPICE3 dynamically allocates memory to relevant entries only. Instead of storing the matrix as a two-dimensional array, a linked list which stores entry values and matrix indices is used to store the same information. Additional code is necessary to manipulate the matrix and to solve equations. Not only do sparse matrices save memory, but LU decomposition and matrix equation solving proceed more quickly due to the elimination of multiply by zero operations.

All of the methods discussed above have been included in TRANSIM and are explored in several different permutations in the following examples. In all cases, the simulations were performed on a DECstation 5000.

A.2.1 Single Tone Example

To demonstrate the effectiveness of the different techniques discussed above, we consider the distributed amplifier shown in Fig. 4. First consider Newton's method without any enhancements, i.e., without sparse matrix routines or the block Jacobian and chord methods. The most time consuming part of the iterative process is building the Jacobian and finding $\mathbf{J}^{-1}({}^i\mathbf{x})$. The unknown phasor quantities to be found are the MESFET terminal voltages and the current through each voltage source for a total of 9 unknowns. A 10 dBm signal at 4Ghz is applied at the input and we consider DC and 7 harmonics in the solution. Because the real and imaginary parts of each quantity are considered separately, the Jacobian is of dimension $(2\cdot8\cdot9)\times(2\cdot8\cdot9)$. The conversion of a vector of time domain derivatives to the frequency domain requires a matrix multiplication followed by an NFFT. For each MESFET, there are four such vectors, yielding twelve matrix multiplications and transforms.

First the single tone analysis was performed without any enhancements. Next, the block Newton method was used for up to 8 blocks. The chord method was then used both by itself and

Table 1: Run times in seconds for single tone analysis of the distributed amplifier.

	No Chord		Chord	
Number of Blocks	Dense	Sparse	Dense	Sparse
1	47.6	21.4	24.7	5.4
2	35.9	12.2	29.1	7.3
4	35.1	9.4	19.4	6.5
8	43.4	10.9	17.5	5.9

Table 2: Memory usage in megabytes for single tone analysis of the distributed amplifier.

	No Chord		Chord	
Number of Blocks	Dense	Sparse	Dense	Sparse
1	3.9	2.3	3.8	2.1
2	4.4	2.3	4.3	2.1
4	4.1	2.3	3.9	2.1
8	4.0	2.3	3.8	2.1

with the block method. Resulting run times and memory usage are shown in Table 1 and Table 2. Run times given include parsing the netlist, initializing data structures, and printing output as well as the time required for analysis.

Sparse matrix routines were then used for single tone analysis. Both the block Newton and chord methods were also used with sparse matrices as described above. Resulting run times and memory usage are shown in Table 1 and Table 2.

The results from each of the permutations of solution methods are the same.

B Transient analysis of quasioptical systems

This work is not complete due to inadequate modeling of the active devices and poor integration of the various tools that have been developed to model the quasi-optical system. What is presented here is preliminary results. We expect to complete the tool integration in the subsequent commercialization activities.

The first target was simulation of the quasi-optical open-cavity oscillator shown in Figure 1. IMPATT diodes were used as the active elements and hence the work described in Section 2.6 on IMPATT diode modeling. Measured and simulated driving point impedances are shown in Figure 5. The modeling strategy is shown in Figure 6.

With just one diode the spectrum of the oscillating signal is shown in Figure 7(a) in the cavity (solid line) and in free space (dashed line). Also shown at the top of the diagram is the measured driving point impedance showing a number of mode resonances. The IMPATT diode was operated in pulsed mode with the measured characteristics shown in Figure 7(b) were obtained. The speckled diagram is the RF envelope while the larger pulse is the profile of the current waveform. Clearly is takes considerable time for oscillations to be established. The best

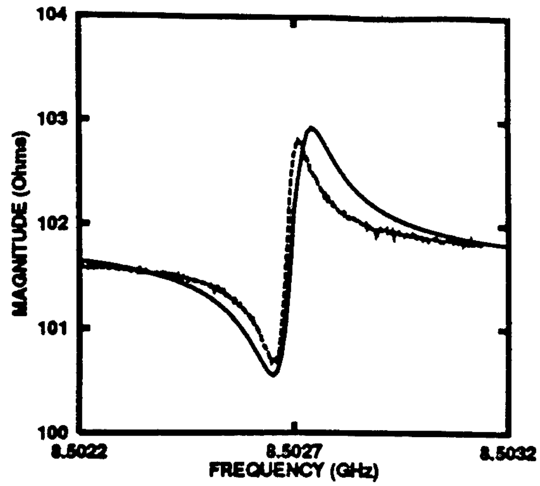


Figure 5: Driving point impedance of a patch antenna in a quasi-optical cavity: solid line is calculated; dashed line is measured.

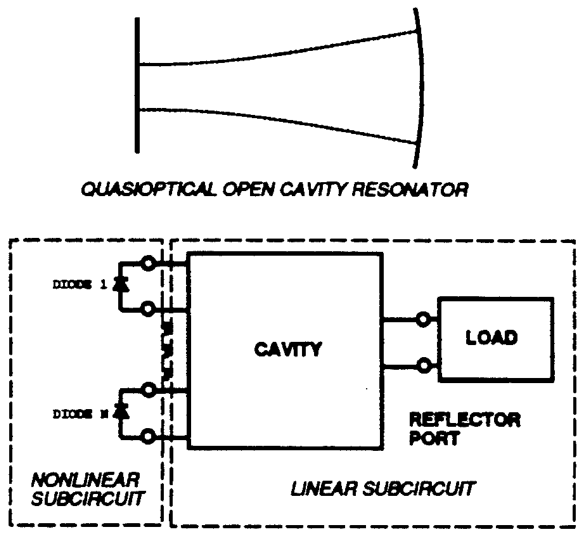


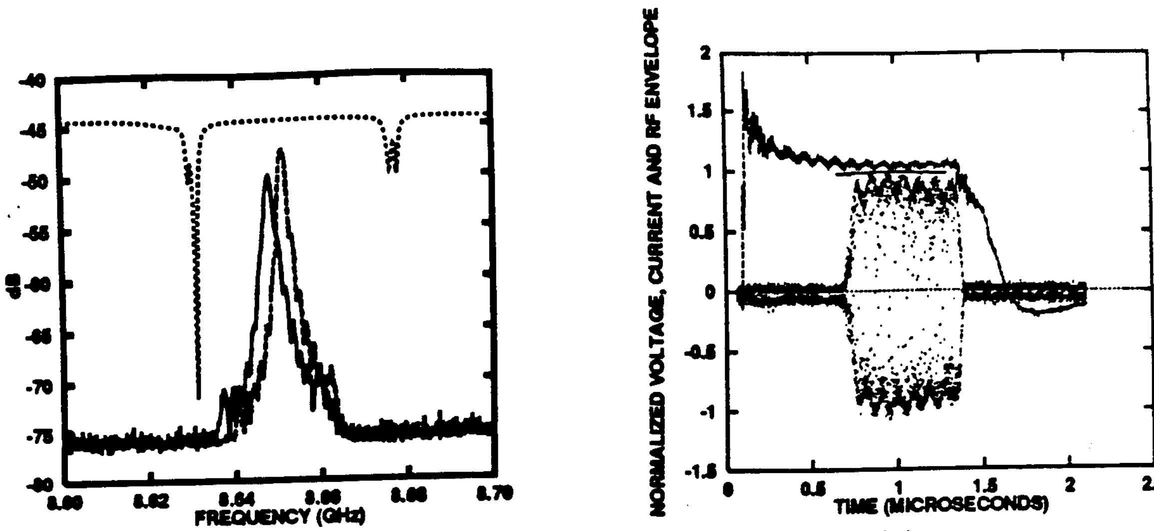
Figure 6: Quasi-optical system modeling.

simulations to date are shown in Figure 8 showing that the correct form of the response was obtained but quantitative agreement is lacking largely due to the tool integration problems cited above.

C Large Signal IMPATT Modeling

C.1 Large Signal IMPATT Operation

- Oscillation is possible with IMPATT diodes because of a negative differential resistance provided by a phase difference between voltage and current.
- For a reverse-biased IMPATT operating under large-signal conditions, there are two delay mechanisms which provide the phase difference.



(a) (b)

Figure 7: Measured spectrum and transient response of a pulsed IMPATT diode power combiner in a Fabry Perot cavity: (a) spectrum, solid line no-cavity, dashed line in-cavity; (b) transient response.

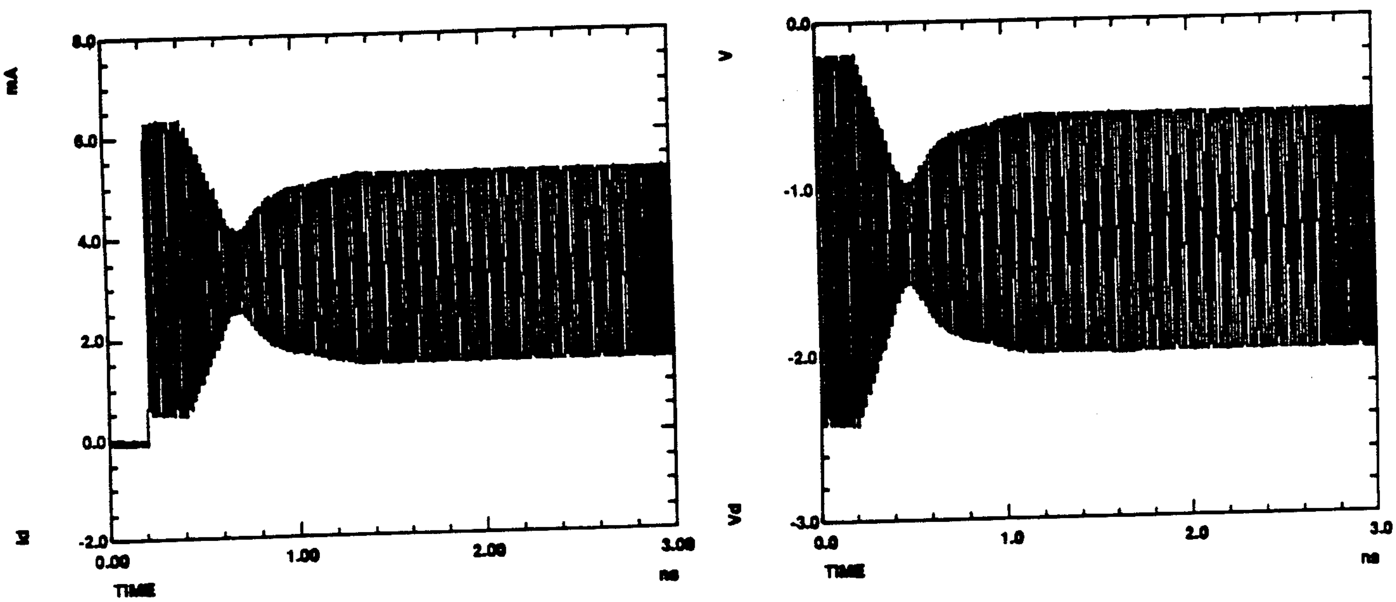


Figure 8: Simulated transient response of an IMPATT diode power combiner in a Fabry Perot cavity using a simple canonical model of the cavity.

The avalanche process

The time required for injected charge to traverse the drift region

- For numerical simplicity, several assumptions must be made for large signal analysis:

  The drift velocity is the same for holes and electrons, and that velocity is equal to the saturation velocity.

  Ionization rates are the same for holes and electrons.

  The ionization coefficient α can be written as

$$\alpha(E) = \alpha_1 e^{-\beta_1/E(x)}.$$

- With the assumption of equal hole and electron velocities and ionization rates, the continuity equations lead to the Read equation:

$$\frac{\tau_a}{2}\frac{\partial i_a}{\partial t} = \left(\int_0^{x_A} \alpha dx - 1\right)(i_a + I_0) + I_s \tag{4}$$

- For simplicity (at least for now), assume linear capacitance for both the avalanche and drift regions.
- Only the ionization integral has a voltage dependence which must be solved in the time domain, so we can solve everything else in the frequency domain.
- Since $\frac{\partial}{\partial t}$ translates to multiplication by $j\omega$ in the frequency domain, we can write the equations as shown below.

Frequency Domain Equations:

$$\begin{aligned} j\omega_k(I_a)_k &= \frac{2}{\tau_a}\left[\mathcal{F}\left\{\left(\int_0^l \alpha dx - 1\right)(i_a + I_0)\right\}_k + I_s\delta(\omega)\right] && (5)\\ j\omega_k(I_d)_k &= \frac{2}{\tau_d}((I_a)_k - (I_d)_k) && (6)\\ & && (7) \end{aligned}$$

C.2 Goeller's IMPATT device model

Goeller's model, shown in Fig. 9 is a popular model for an IMPATT diode and was initially investigated for incorporation in to the Harmonic Balance procedure. It was anticipated that this would be a small drain on the project. However this models turns out to be very difficult to incorporate.

Time Domain Device equations:

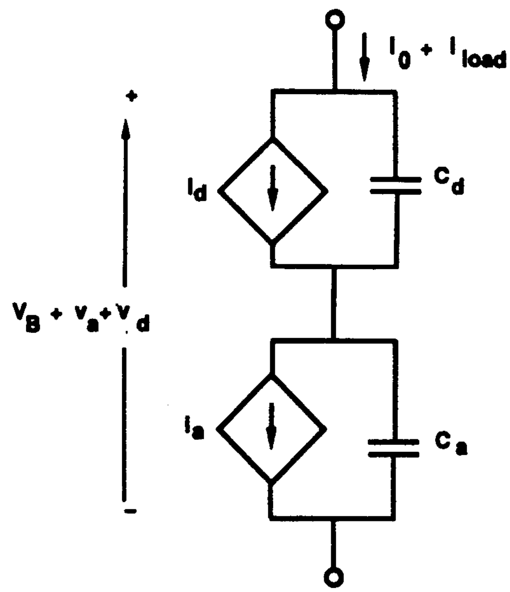


Figure 9: Goeller's IMPATT device model

$$\frac{d}{dt}v_a = \frac{1}{C_a}(i_{load} - i_a) \tag{8}$$

$$\frac{d}{dt}v_d = \frac{1}{C_d}(i_{load} - i_d) \tag{9}$$

$$\frac{d}{dt}i_a = \frac{2}{\tau_a}\left[\left(\int_0^l \alpha dx - 1\right)(i_a + I_0) + I_s\right] \tag{10}$$

$$\frac{d}{dt}i_d = \frac{2}{\tau_d}(i_a - i_d) \tag{11}$$

$$\tag{12}$$

Where

$$\alpha(E) = \alpha_1 e^{-\beta_1/E(x)} \tag{13}$$

and the electric field, $E(x)$ is given by

$$E(x) = \begin{cases} \sqrt{\frac{2qN_1}{\epsilon}}\sqrt{V_B + v_a + v_d} - \frac{qN_1}{\epsilon}x & x \leq b \\ \sqrt{\frac{2qN_1}{\epsilon}}\sqrt{V_B + v_a + v_d} - \frac{qN_1 b}{\epsilon} - \frac{qN_2}{\epsilon}(x-b) & x > b \end{cases} \tag{14}$$

This mathematical model is difficult to implement principally because the time derivative of the avalanche current depend on the avalanche current. This equation must be satisfied at each simulation iteration. The the following section we discuss enhancements to the Goeller model to make it more suitable for Harmonic Balance analysis.

C.3 Solving the Frequency Domain Avalanche Current Equation

$$j\omega_k I_{ak} = \frac{2}{\tau_a}\left[\mathcal{F}\left\{\left(\int_0^l \alpha dx - 1\right)(i_a(t) + I_0)\right\}_k + I_s\delta(\omega)\right] \tag{15}$$

Let $b(t) = \int_0^l \alpha dx - 1$ and $y(t) = b(t)(i_a(t) + I_0)$.
Then the avalanche current equation becomes:

$$j\omega_k I_{ak} = \frac{2}{\tau_a}\left[\mathcal{F}\left\{y(t)\right\}_k + I_s\delta(\omega)\right] \tag{16}$$

If we know $b(t)$ and $i_a(t)+I_0$ in the time domain, then we can use Chang's Arithmetic Operator Method (AOM) to find the spectral content of $y(t)$. This is written

$$\mathbf{y} = \mathbf{T_b}\mathbf{i} \tag{17}$$

where $\mathbf{T_b}$ is the "spectral transform matrix" of $b(t)$ and bold type indicates spectral quantities.

The avalanche current equation then can be solved for $K+1$ frequency components as a set of $K+1$ linear equations.

$$\begin{bmatrix} 0 \\ j\omega_1 I_{a1} \\ \vdots \\ j\omega_k I_{aK} \end{bmatrix} = \begin{bmatrix} B_{00} & B_{01} & \dots & B_{0K} \\ B_{10} & B_{11} & \dots & B_{1K} \\ \vdots & & & \\ B_{K0} & B_{K1} & \dots & B_{KK} \end{bmatrix} \cdot \begin{bmatrix} I_{a0} + I_0 \\ I_{a1} \\ \vdots \\ I_{aK} \end{bmatrix} + \begin{bmatrix} I_s \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{18}$$

So for each iteration of the harmonic balance simulation, we must solve a $(K+1) \times (K+1)$ system of equations.

C.4 Harmonic Balance implementation of Goeller's IMPATT device model

- For simplicity (at least for now), assume linear capacitance for both the avalanche and drift regions.
- Only the ionization integral has a voltage dependence which must be solved in the time domain, so we can solve everything else in the frequency domain.
- Since $\frac{\partial}{\partial t}$ translates to multiplication by $j\omega$ in the frequency domain, we can write the equations as shown below.

Frequency Domain Equations:

$$j\omega_k(I_a)_k = \frac{2}{\tau_a}\left[\mathcal{F}\left\{\left(\int_0^l \alpha dx - 1\right)(i_a + I_0)\right\}_k + I_s\delta(\omega)\right] \tag{19}$$

$$j\omega_k(I_d)_k = \frac{2}{\tau_d}((I_a)_k - (I_d)_k) \tag{20}$$

$$\tag{21}$$

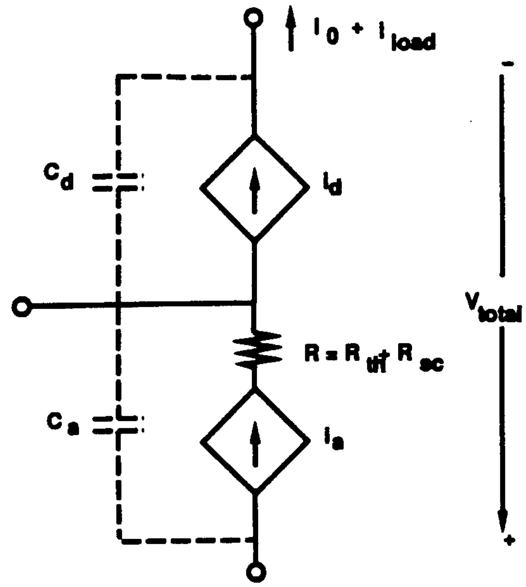


Figure 10: Harmonic balance compatible Goeller's IMPATT device model

The result is an harmonic balance compatible Goeller's IMPATT device model as shown in Figure10.

C.5 Including Effective Resistance in the Frequency Domain Avalanche Current Equation

$$j\omega_k I_{ak} = \frac{2}{\tau_a}\left[\mathcal{F}\left\{\left(\int_0^l \alpha dx - 1\right)(i_a(t) + I_0)\right\}_k + I_s\delta(0)\right] \tag{22}$$

To solve the above equation when the resistance due to thermal effects and space charge is included, an iterative approach must be used.

At each harmonic balance iteration, we have the spectral content of voltage v_a from the vector of unknowns $^i\mathbf{x}$.

- Guess the values of $v_x(t)$, where $v_x(t) = v_a(t) - (i_a(t) + I_0)R$.
- Using the $v_x(t)$ values, find $i_a(t) + I_0$ from the procedure discussed previously.
- Use these values in the equation $v_a(t) = v_x(t) + (i_a(t) + I_0)R$ to obtain an error function.
- Minimize the error function.

C.6 Analytical Solution of the Ionization Integral

We can rewrite the expression for electric field:

$$E(x) = \begin{cases} A - Bx & x \leq b \\ C - Dx & x > b \end{cases} \tag{23}$$

where

$$
\begin{array}{rcl}
A & = & \sqrt{\frac{2qN_1}{\epsilon}}\sqrt{V_B + v_a + v_d} \\
B & = & \frac{qN_1}{\epsilon} \\
C & = & A - \frac{q(N_2 - N_1)b}{\epsilon} \\
D & = & \frac{qN_2}{\epsilon}
\end{array}
\tag{24}
$$

From Maple, the integral

$$\int_0^l e^{-\beta_1/E(x)}dx \tag{25}$$

is evaluated as

$$
\begin{aligned}
& \frac{1}{B}\left((-A+Bl)e^{\frac{-\beta_1}{A-Bl}} + \beta_1 E_1(\beta_1/(A-Bl)) + Ae^{\beta_1/A} - \beta_1 E_1(\beta_1/A)\right) \\
+ & \frac{1}{C}\left((D-Cl)e^{\frac{-\beta_1}{D-Cl}} - \beta_1 E_1(\beta_1/(D-Cl))\right)
\end{aligned}
\tag{26}
$$

where E_1 is the exponential integral.

C.7 Parameter Extraction of the IMPATT Model

To use the equations in simulation, we need values for α_1 and β_1 which predict currents and voltages which match measured data. The process of extracting these values requires the formulation of some error function and the use of an optimization method to minimize this function. A hybrid optimization method which uses tree annealing to find "valleys" of the error function and gradient descent to explore these valleys quickly is used to find α_1 and β_1.

The first attempt to find these parameters involved using the Read equation at DC to write current as a function of voltage

$$I_0 = \frac{I_s}{\int_0^l \alpha_1 e^{-\beta_1/E(x)}dx - 1} \tag{27}$$

where the measured current is I_0 and the measured voltage is present in $E(x)$. Unfortunately, only one measured (V_0, I_0) pair at a time could be matched. This is because the equation as written above does not take into account thermal and space charge resistances which are present in the measurements.

To account for the resistance present in the measurements, the electric field can be rewritten

$$E(x) = K_1\sqrt{V_0 - I_0 R} - K_2 x \tag{28}$$

From the Read equation we can write α_1 as a function of β_1

$$\alpha_1 = \frac{I_s/I_0 + 1}{\int_0^l e^{-\beta_1/E(x)}dx} \tag{29}$$

And the error function then becomes the difference between calculated α_1 values for different (I_0, V_0) pairs. This method of extraction has found many different (α_1, β_1) pairs which satisfy more than one (V_0, I_0) pair at a time.